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The hydrodynamic squeeze film lubrication of the ankle joint

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Abstract: The main aim of the study presented in this paper is to determine the characteristics of synovial fluid film region by modeling the human ankle joint to obtain an analytical expression for the pressure distribution, load carrying capacity, coefficient of friction and reduction of synovial film thickness with time of approach. Thus in order to reach a comprehensive analysis of the human ankle joint lubrication, variable behavior of synovial fluid during different time of joint activities and the influences of articular cartilage has to be taken into account. The behavior of the synovial fluid has been assumed to be isothermal, shear thinning and non- Newtonian couple stress fluid. The model of ankle joint has been taken geometrically and kinematically as a partial porous journal bearing under the action of hydrodynamic squeeze film lubrication. Typical geometrical and physical values of the ankle joint were acquired from measured values reported in literature. The problem of ankle joint lubrication has been solved numerically for various couple stress fluid parameters together with effect of varying the porosity of the articular cartilage. It has been shown , in the presence of porous articular cartilage, and by assuming the synovial fluid to be a non- Newtonian shear thinning couple stress fluid that the , pressure distribution, load carrying capacity, synovial film thickness with time of approach increased and a reduction coefficient of friction in the hydrodynamic squeeze action in the ankle joint resulted.

Keywords: Ankle Joint, Synovial Fluid, Couple Stress Fluid, Hydrodynamic Lubrication, Human Joint Lubrication

1. Introduction

The lubrication of human synovial joints has received some attention the last 30 year. The ankle joint is a very important joint because it is a high load-carrying joint in the human body. It is an essential element in maintaining posture and equilibrium of the body. Without ankles we cannot carry out important physical activities such as standing, walking or any type of sports. Thus, it is essential to study and understand the joint lubrication mechanism.

The squeeze film phenomenon arises when the two lubricating surfaces move towards each other in the normal direction and generate a positive pressure and hence support a load. This is due to the fact that a viscous lubricant present between the two surfaces cannot be instantaneously squeezed out when the two surfaces move towards each other and this action provides a cushioning effect in bearings. The squeeze film lubrication phenomenon is observed in several applications such as gears, bearings, machines tools, rolling elements and automotive engines [1], dampers and human joints [2].

The ankle joint consists of three bones namely, the tibia, fibula, and talus. The tibia and fibula form the mortise (socket) into which the talus fits, forming the hinge joint. The talus constitutes the link between the leg and the rest of the foot. The superior articular surface is covered with cartilage for articulation with the medial malleolus portion of the tibia, the inferior surface of the tibia, and the medial surface of the fibula [3], shown in Figure (1).

Articular cartilage is the bearing material that lines the ends of the bones of synovial joints. Its primary function is to reduce friction and wear at the articulations of the musculoskeletal system. The tribological properties of cartilage are intimately related to its structure and mechanical properties. The modes of lubrication in cartilage extend beyond the traditional mechanisms of fluid-film or boundary lubrication. The purpose of this review is to describe the salient properties of articular cartilage necessary to understand the unique biotribology of diarthrodial joints [4].

Cartilage is a white connective tissue which is synthesized and maintained by cells called chondrocytes .In human joints, the thickness of the articular cartilage layer varies from 0.5 to 1.5 mm in upper extremity joints, such as the hand and the shoulder ,and from 1 to 6 mm in lower extremity joints, such as the hip, knee, and ankle [4]. Under normal conditions, articular cartilage provides low friction and wear over a life span. It is a highly hydrated tissue, with a porosity varying from 68 to 85 per cent in adult joints [4, 5].

Synovial fluid is secreted by synovial lining cells. It plays a very important role in synovial joints. It occupies the joint cavity and lines the synovial joint, providing nutrients and removing catabolic products [6, 7].

Synovial fluid is essentially a dialysate of blood plasma with the addition of long chain polymer (hyaluronic acid). It is hyaluronic acid that is responsible for the viscosity of synovial fluid and if it is all removed by acetic acid precipitation, the viscosity of the fluid reduces to the water [8].Normal healthy synovial fluid is highly non-Newtonian [8,9, 10, 11, 12], such that the viscosity reduces markedly with the increase of shear rate. In osteoarthritis the viscosity is reduced particularly at the low shear rates while in rheumatoid arthritis the viscosity is reduced even more .This means that with increasing severity of disease the viscosity reduces and becomes less dependent on the rate of shear.

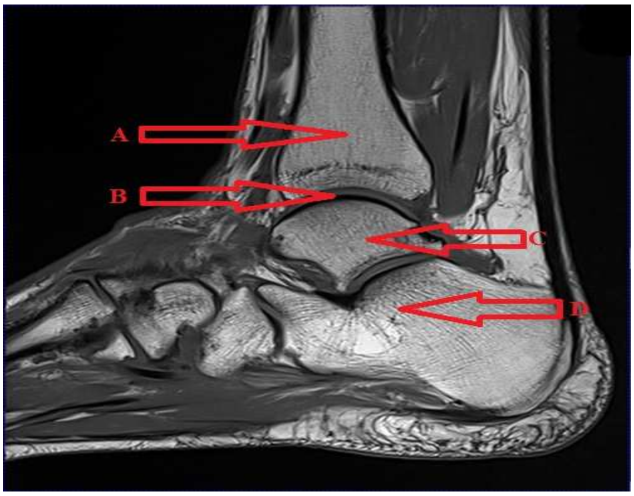

Figure (1) *show the human ankle joint (A) tibia bone (B) synovial film thickness (C) talus bone (D) calcaneus bone [30].*

2. Human ankle Joint Geometry Model

The human ankle joint can be represented kinematically and geometrically by cylindrical surfaces[9,13,14], where the motion is allowed only in the sagittal plane [9, 14, and 15]. The coupling stress model is assumed by two infinite rigid circular cylinders (subchondral bone) in the internal contact (a cylinder encased in a cylindrical cavity), covered with thin layer (articular cartilage) of uniform thickness, the lower (talar) articular surface is supposed stationary while the upper (tibial) surface is assumed to have pure squeeze motion.

The synovial fluid is considered as mixture of two incompressible fluids: viscous (hyaluronic acid-protein macromolecular complex) and ideal (water and small solutes), only the ideal passing across pores of the articular surface, thus enabling a synovial gel film formation. According to this asymptotic model, the filtration by cartilage is intensive, the fluid film quickly becomes depleted, and a synovial gel layer develops over the greater part of the contact in the step-loaded joint. The gel can serve as a boundary lubricant if a sliding motion follows before a fresh synovial fluid gets into the contact. [14, 15, 16].

The long chain hyaluronic acid (HA) molecules found as polar additives in synovial fluid are characterized by two material constants (η) and (μ). Synovial fluid usually exhibits a non- Newtonian shear thinning behavior. However, under high shear rates, the viscosity of synovial fluid approaches a constant value not much higher than that of water [7]. Therefore a Newtonian lubricant model has often been used for synovial fluid in lubrication modeling [18] and in certain pathological cases it becomes Newtonian. The similar material constants are also found in Stokes [19] couple-stress fluid. The simplest generalization of the classical theory of fluids by Stokes provides the macroscopic description of the behavior of fluids containing a substructure such as lubricants with polymer additives. This has given the motivation to model synovial fluid as couple-stress fluid in human joints. [9, 10, 11, 20, 21] The coordinates (x) and (y) used for the description of the fluid film pressure and the movement occurs in the (x–y) plane, as shown in Figure (2). It is assumed that the ankle joint is loaded in the (y) direction in the plane of symmetry by a load W (t) [N / m], occurring during standing and steady walking.

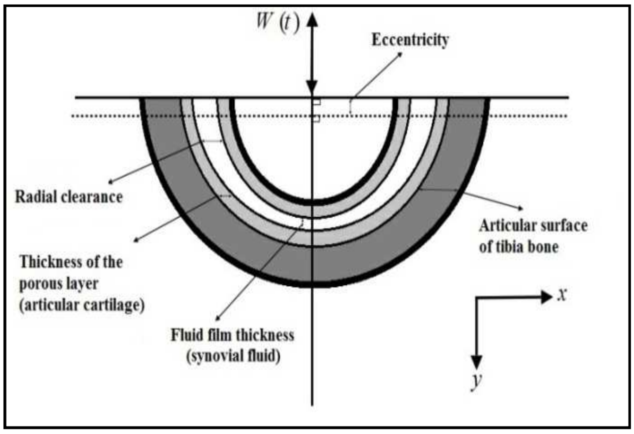

Figure (2) *shows the physical and geometry configuration of the ankle joint (partial porous journal bearing).*

3. Governing Equation

The basic field equations of the micropolar fluid were developed by Eringen [23] and Stokes [19] and later adopted by Prakash and Sinha[22]. The basic equations for the flow of micro polar fluid in the film region in vectorial form are [23].

$$\rho\frac{dv}{dt} = -\nabla p + \mu\nabla^2\vec{V} - \eta\nabla^4\vec{V} \tag{1}$$

The steady-state laminar equation of flow of the fluid film region in Cartesian coordinates is the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{2}$$

Under the usual assumptions of fluid film lubrication applicable to thin films ,the equation of motion of an incompressible couple stress fluid within the film region[9, 10,24] the equation(1)becomes:-

$$\frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial^4 u}{\partial y^4} \tag{3}$$

The ratio $(\frac{\eta}{\mu})$ is a dimensional square length and hence characterizes the chain length.

$$l = \sqrt{\frac{\eta}{\mu}} \tag{4}$$

The flow of couple stress fluid in a porous matrix is governed by the modified Darcy law, which accounts for the polar effects [24].

$$q^{\rightarrow} = \frac{-\phi}{\mu(1-\beta)}\nabla p^* \tag{5}$$

as $q^{*\rightarrow} = (u^*, v^*)$ and $\beta = \frac{(\frac{\eta}{\mu})}{\phi} = \frac{l^2}{\phi}$

$$-v_\theta - \frac{\phi H_0}{\mu(1-\beta)}(\frac{\partial^2 p}{\partial x^2}) = -\frac{\partial}{\partial x}\int_0^h u\ dy \tag{6}$$

The relevant boundary conditions for the velocity components are:

i. At the porous journal surface y = 0

$$u = 0,\ \frac{\partial^2 u}{\partial y^2} = 0,\ \text{and}\ V = v^* \tag{7}$$

ii. At the boundary surface y = h

$$u = 0,\ \frac{\partial^2 u}{\partial y^2} = 0\ \text{and}\ V = -v_\theta \tag{8}$$

The solution of equation (3) subject to the boundary conditions (6) and (7) and applying sine and cosine hyperbolic expressions to get final form of velocity in Cartesian coordinates as :-

$$u(x,y) = \frac{1}{2\mu}\frac{\partial p}{\partial x}\{y(y-h) + 2l^2[1 - \frac{\cosh(\frac{2y-h}{2l})}{\cosh(\frac{h}{2l})}]\} \tag{9}$$

Integrating equation (2) across the fluid film and utilizing boundary conditions (7) and (8) and expressions in (6) and (9) in the modified Reynolds equation as applicable to squeeze action is obtained in the form

$$\frac{\partial}{\partial x}\{[f(h,l) + (\frac{12\phi H_0}{(1-\beta)})]\frac{\partial p}{\partial x}\} = -12v_\theta\mu \tag{10}$$

Where function $f(h,l)$ is given as:-

$$f(h,l) = h^3 - 12l^2h + 24l^3\tanh(\frac{h}{2l})$$

3.1. Squeeze Film Pressure of the Porous Journal Bearing Representing the Ankle Joint

Introducing the following non-dimensional quantities of the governing equation of pressure for neat presentation as follows:-

$$p^* = \frac{pC^2}{\mu R^2(d\varepsilon/dt)},\ h^* = \frac{h}{C} = 1 - \varepsilon\cos(\theta),$$

$$\theta = \frac{X}{R},\ l^* = \frac{l}{C},\ \phi^* = \frac{\phi}{C^2},\ H^* = \frac{H}{C}$$

$$R^* = \frac{R}{C},\ \psi = \frac{kH}{C^3},\ \varepsilon = \frac{e}{C}$$

Applying the above dimensionless equations in equation (10) .Thus the final dimensionless form of the modified Reynolds equation becomes:

$$\frac{\partial}{\partial\theta}\{[f(h^*,l^*) + \frac{12H^*\phi^*}{(1-\beta)}]\frac{\partial p^*}{\partial\theta}\} = -12\cos(\theta) \tag{11}$$

The boundary conditions for the fluid film pressure are:

$$p^* = 0 \quad at\ \theta = \pm\frac{\pi}{2} \tag{12}$$

$$\frac{dp^*}{d\theta} = 0 \quad at\ \theta = 0 \tag{13}$$

Integrating equation (11) with boundary conditions (12), (13) and inserting the two limits (θ, $\frac{-\pi}{2}$) to obtain final form of pressure distribution:

$$p^*(\theta) = \frac{-3(\pi^2 - 4\theta^2)}{2((-1+\varepsilon)^3 - 12(-1+\varepsilon)l^2 - (\frac{12\phi^* H^*}{(1-\beta)}) - 24l^3\tanh(\frac{1-\varepsilon}{2l}))} \tag{14}$$

3.2. Load Carrying Capacity of the Porous Partial Journal Bearing Representing Ankle Joint

The load carrying capacity of the porous bearing in a squeeze action is obtained by integrating the film pressure equation acting on the surface of the journal shaft. The load carrying capacity is there given by:-

$$W = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p(\theta).R.\cos(\theta) \tag{15}$$

Let the dimensionless load carrying capacity be:

$$W^* = \frac{WC^2}{\mu R^3(\frac{d\varepsilon}{dt})} \tag{16}$$

$$W^* = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} P^*(\theta)\cos(\theta)\,d\theta \tag{17}$$

The terms $p^*(\theta)$, $p^*(\theta).\cos(\theta)$ is difficult to deal with by usual mathematical methods, and it cannot be obtained by direct integration. Thus the use of a solution method such as (power series, Gaussian Quadrature and Simpson method) is appropriate .In this research the power series was used and both terms have been substituted with the well fitting approximate function which has been substituted with power series , then integrating the output resulting from the power series to get the dimensionless load. All mathematical analyses and output resulting curves were done by computer program called "wolfram Mathematica (8.0)". Then equation (17) becomes:-

$$W^* = \frac{-\pi^3(1280-32\pi^2+\pi^4)}{1280((-1+\varepsilon)^3-12(-1+\varepsilon)l^2-(\frac{12\phi^* H^*}{(1-\beta)})-24l^3\tanh(\frac{1-\varepsilon}{2l}))} \tag{18}$$

3.3. Squeeze Time-film Thickness Relationship

The most important characteristics of the squeeze film bearings is the squeeze film time, i.e., the time required for reducing the initial film thickness (h^*) to (h^*_o) (minimum film thickness).

The response time of the squeeze film is one of the significant factors for the deep understanding of biological bearings. The response time is the time that will elapse for a squeeze film to be reduced to some minimum permissible level. , The film thickness at any time (t) can be obtained by integrating equation (18) for the given load [22].

$$t^* = \int_{h^*}^{1} W^* d\varepsilon \tag{19}$$

The detailed result of equation (19) can be seen in appendix (A).

3.4. Coefficient of Friction

The friction force can be obtained by integrating the shear stress around the journal surface, as follows [19].

$$\tau = \mu(\frac{\partial u}{\partial y})_{y=h} - \eta(\frac{\partial^3 u}{\partial y^3})_{y=h} \tag{20}$$

$$\tau = \mu(\frac{U}{h} + \frac{h}{2\mu}\frac{\partial p^*}{\partial x}) \tag{21}$$

Thus, the equation of friction force in a dimensionless form is [28, 29]:-

$$F^* = \int_0^1 (\frac{1}{h^*} + \frac{h^*}{2}\frac{\partial p^*}{\partial \theta})\,d\theta \tag{22}$$

Substitute for (p^*) from equation (14) in equation (23) obtain the dimensionless friction force. The coefficient of friction can be calculated from the relation [28, 29]:-

$$C_f = \frac{F^*}{W^*} \tag{23}$$

The result of equation (23) can be seen in appendix (B).

4. Results and Discussion

4.1. Result

The lubrication of the human ankle joint object is very difficult if not impossible to study in vivo. For this reason the need for a numerical study arises, and the selection of values for a good estimation to get the best analysis results. The parameters were chosen based on previously selected values use in research, specializing with problem to reach to practical parameters that are close to reality such as the dimensions of ankle joint, articular cartilage thickness and permeability of porous region. The selected parameter values are listed in table (1)

Table (1) *the numerical values of the parameters involved [9, 13, 25].*

Parameters	Numerical values	units
Radial Clearance (C)	1.2×10^{-3}	(m)
Dimensionless couple stress length (l^*)	0.1-0.7	-
Eccentricity ratio parameter (ε)	0.1-0.6	-
Permeability of the cartilage matrix (ϕ)	10^{-18}	(m^2)
Tibia and Talus cartilage thickness	1.2×10^{-3}	(m)
Viscosity of synovial fluid(μ)	0.01	Pa.s

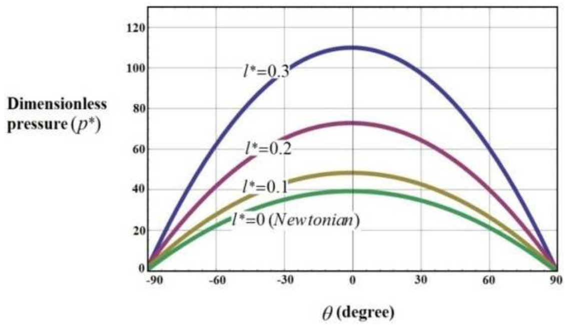

Figure (3) *shows the variation of dimensionless pressure (p^*) with angle (θ^o) for different couple stress length parameters (l^*),* $\phi = 10^{-18}\ m^2$ *and* ε *=0.3.*

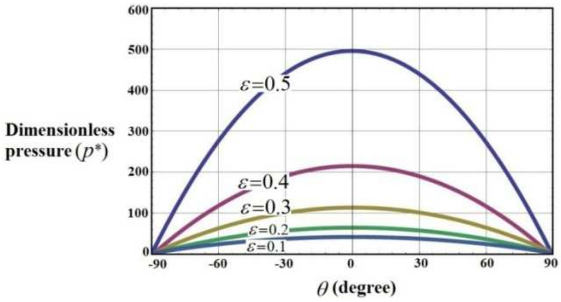

Figure (4) *shows the variation dimensionless pressure (p^*) with angle (θ^o) for different eccentricity ratios (ε),* $l^* = 0.3$ *and* $\phi = 10^{-18}\ m^2$.

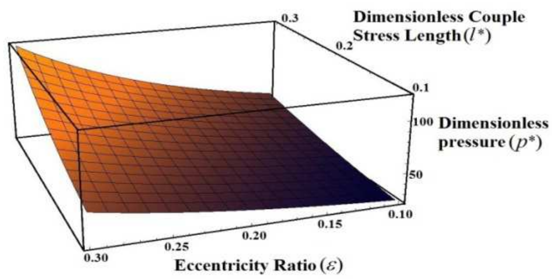

Figure (5) *shows the variation of dimensionless pressure (p^*) with couple stress length parameters (l^*) and eccentricity ratios (ε), for* $\theta^o = 0$ *and* $\phi = 10^{-18}\ m^2$.

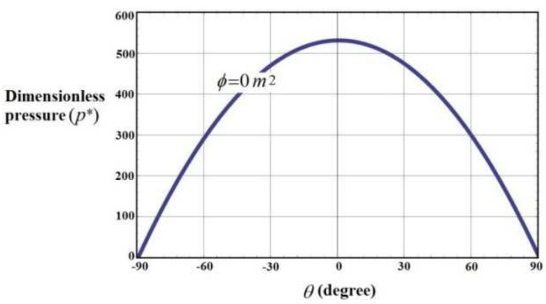

Figure (6) *shows the variation dimensionless pressure (p^*) with angle (θ^o) at Permeability value of the cartilage matrix* $\phi = 0\ m^2$ *with* $l^* = 0.3$, $\varepsilon = 0.5$.

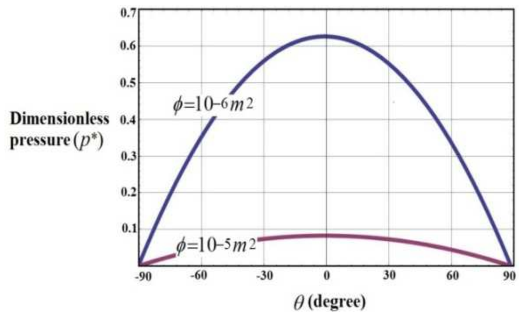

Figure (7) *shows the variation dimensionless pressure (p^*) with angle (θ^o) for different Permeability value of the cartilage matrix (ϕ) with* $l^* = 0.3$, $\varepsilon = 0.5$.

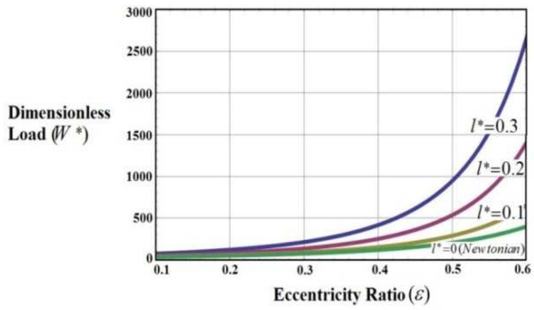

Figure (8) *shows the variation dimensionless load (W^*) with eccentricity ratio (ε) for different couple stress length parameters (l^*),* $\phi = 10^{-18}\ m^2$.

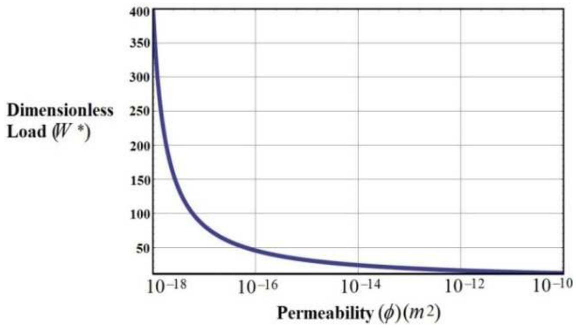

Figure (9) *shows the variation dimensionless load (W^*) with different Permeability value of the cartilage matrix (ϕ) at* $\varepsilon = 0.6$, $l^* = 0$ *(Newtonian)*

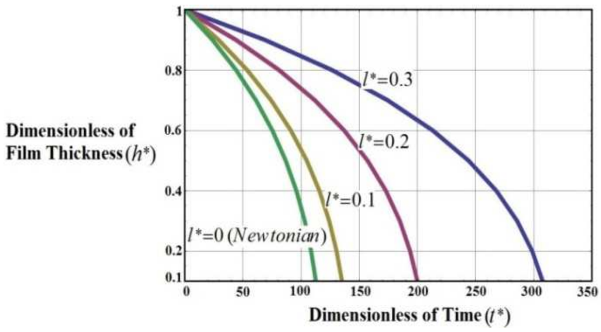

Figure (10) *Shows the variation dimensionless Squeeze Time (t^*) with dimensionless film thickness (h^*) for different couple stress length parameters.*

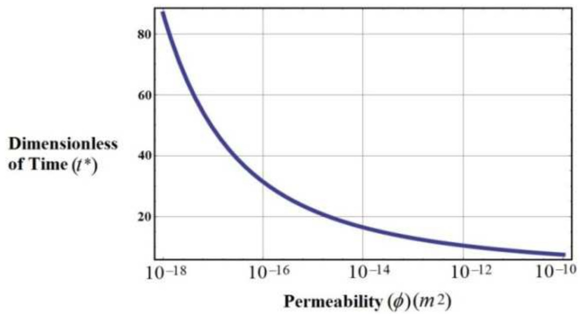

Figure (11) *Shows the variation dimensionless Squeeze Time (t^*) with different Permeability value of the cartilage matrix at $h^*=0.5$, $l^*=0$ (Newtonian)*

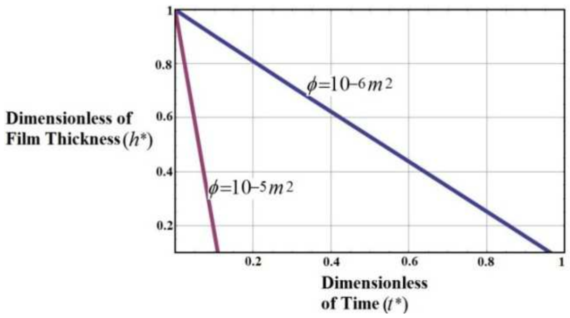

Figure (12) *shows the variation dimensionless Squeeze Time (t^*) with dimensionless film thickness (h^*) for different values of permeability with $l^*=0.3$*

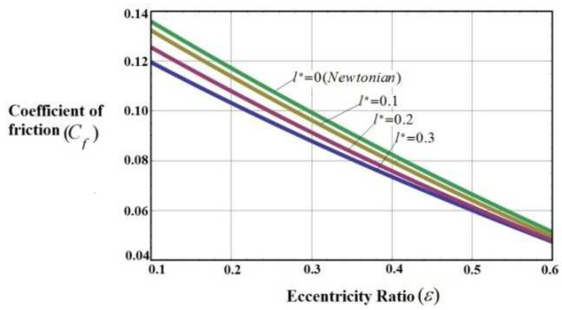

Figure (13) *shows the variation of the dimensionless coefficient of friction (C_f) with eccentricity ratio (ε) for different couple stress length parameters (l^*), $\phi = 10^{-18}\ m^2$*

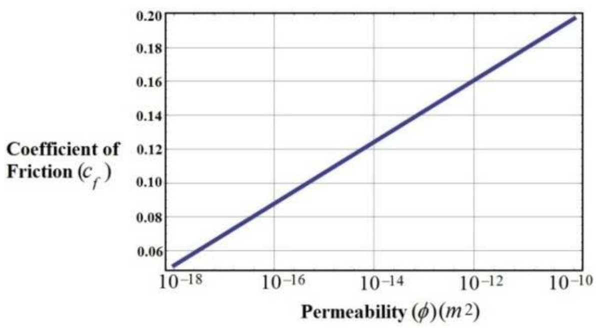

Figure (14) *shows the dimensionless coefficient of friction (C_f) with different Permeability value of the cartilage matrix (ϕ) at $\varepsilon=0.6$, $l^*=0$ (Newtonian)*

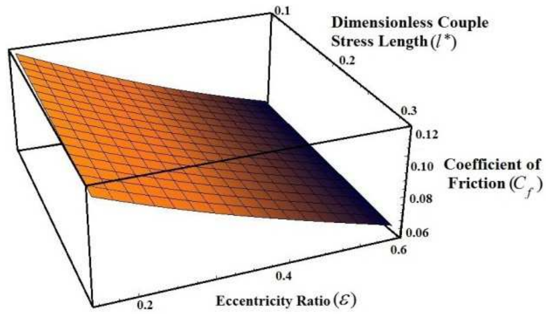

Figure (15) *shows the variation of the dimensionless coefficient of friction (C_f) with eccentricity ratio (ε) for different value of the couple stress length parameters (l^*)*

4.2. Discussion

The model, on the basis of the Stokes micro-continuum theory, takes into account the non- Newtonian nature of the synovial fluid considering the effects of couple stresses on the squeeze film lubrication of the ankle joint. The film pressure was solved using a modified Reynolds equation for a non-Newtonian couple stress fluid. It was applied to predict the pressure distribution and load-carrying capacity for lubricated system of human ankle joint in squeeze action. As the value of couple stress parameter (l^*) for the fluid used approaches zero as shown in figures (3),(4)and (8), the bearing characteristics predicted by the present analysis approach the Newtonian-lubricant case. This is the agreement with results reported [9, 14, 24]. According to the results obtained, the influence of couple stress effects on the bearing characteristics is physically significant. It found through that, the effect of large permeability values reduce the dimensionless pressure distribution, dimensionless load-carrying capacity, time of approach and increase the coefficient of friction as compared to the corresponding normal size of porosity.

In contrast, the lowest pore size leads to the increase in the amount of load and this explains the influence of osteoarthritis disease on the efficiency of ankle joint function, where the cartilage is less stiff in both compression and shear, and fluid flows more easily through the tissue in joints with osteoarthrosis.

This implies greater displacement of osteoarthrotic cartilage than normal (decreased stiffness) and a greater rate of deformation (increased permeability) [26] as shown in figures (6), (7), (9), (11), (12), and (14).

The inverse proportionality relationship between squeeze time and film thickness has been seen in the journal bearing model of ankle joint lubrication. At the initial time, the film thickness has its maximum value and Vice versa, when time elapses, the film thickness is reduced until reaching a minimum value, as shown in figure (10).The results of coefficient of friction explain the inversely proportional relationship between eccentricity ratio and its value. When the joint moves, the film thickness decreases leading to a decrease in coefficient of friction value between articular cartilages ,as shown in figure(13). This decrease in coefficient of friction depends on a many parameters such as eccentricity ratio (ε), couple stress length parameter (l^*) and permeability of porous articular cartilage. The values obtained in figure (13) are close to the values of reported in reference [27], which specializes in the study of the frictional properties of articular cartilage.

5. Conclusions

On the basis of the results presented, the following conclusions can be drawn:

- The long chain of hyaluronic acid molecules existing in the synovial fluid gives the motivation for assuming the synovial fluid as a Stokes Couple stress fluid.
- The effect of micropolar fluid provides an increased pressure, load carrying capacity, squeeze film time and decrease coefficient of friction as compared to the corresponding Newtonian case.
- The synovial fluid film thickness between two the articular cartilages reduces gradually with continuous application of the load. The degree of this reduction depends on many reasons such as, type of daily activities [standing, walking, running], healthy of synovial fluid and time of applying load on the ankle joint.
- Radial clearance parameter is important factor to determine features of partial journal bearing of ankle joint because of its direct effect on the computational of all physical parameters such as fluid pressure, load applied, coefficient of friction and fluid film thickness variation with squeeze time.
- Joints with normal cartilage give high load capacity and low coefficient of friction as compared with degenerating cartilage (Osteoarthritis), which is a desirable aspect in typical synovial joint lubrication.
- The effect of large porosity parameter in articular cartilage causes reduction in load carrying capacity and time of approach to minimum film thickness as compared with small porous size.

Nomenclature

∇ gradient operator

C radial clearance, m

e eccentricity, m

$\varepsilon = (\frac{e}{C})$ eccentricity ratio.

ϕ Permeability of the cartilage matrix, m^2

$\phi^* = (\frac{\phi}{C})$ dimensionless permeability of the cartilage matrix .

$\psi = \frac{\phi H}{C^3}$ permeability parameter.

μ viscosity of synovial fluid , pa.s

η material constant responsible for couple stress property.

$l = \sqrt{\frac{\eta}{\mu}}$ couple stress parameter , m

$l^* = (\frac{l}{C})$ dimensionless couple stress parameter .

u ,v components of fluid velocity in x, y, directions, respectively , m/s

$\vec{V}$ velocity vector

h synovial film thickness , mm

$h^* = (\frac{h}{c})$ dimensionless film thickness.

H porous layer thickness , mm

$\beta = (\frac{l^2}{\phi})$ ratio of microstructure size to pore size.

p pressure in the porous region , pa

p^* dimensionless pressure .

W load carrying capacity per unit length of the bearing , N/m

W^* dimensionless load capacity .

F^* dimensionless frictional force .

ρ density of synovial fluid , g/cm3

t response time taken by journal centre to move from ε =0 to ε_1 , sec.

t^* dimensionless response time .

R radius of the journal , mm

C_f coefficient of friction.

Appendix (A)

The output result of equation (19)

$$t^* = \frac{-(-1+h)\pi^3(1280-32\pi^2+\pi^4)}{2560(1-12\,l^2+(\frac{12\,\phi^*H^*}{(1-\beta)})+24\,l^3\tanh(\frac{1}{2l})}(9-60\,l^2+288\,l^4$$

$$+h^2(4+24\,l^2-2(\frac{12\,\phi^*H^*}{(1-\beta)}))+5(\frac{12\,\phi^*H^*}{(1-\beta)})-48\,l^2(\frac{12\,\phi^*H^*}{(1-\beta)})+2(\frac{12\,\phi^*H^*}{(1-\beta)})^2$$

$$+h(7-12\,l^2+(\frac{12\,\phi^*H^*}{(1-\beta)}))-4\,l(-1+288\,l^4+h(-1+6\,l^2-(\frac{12\,\phi^*H^*}{(1-\beta)}))$$

$$+h^2(-1+24\,l^2-(\frac{12\,\phi^*H^*}{(1-\beta)}))-(\frac{12\,\phi^*H^*}{(1-\beta)})-6\,l^2(3+4(\frac{12\,\phi^*H^*}{(1-\beta)}))\tanh(\frac{1}{2\,l})$$

$$+12\,l^2(-5+20\,l^2+96\,l^4+h^2(-5+20\,l^2-(\frac{12\,\phi^*H^*}{(1-\beta)}))-(\frac{12\,\phi^*H^*}{(1-\beta)})$$

$$\tanh(\frac{1}{2\,l})^2-4\,l^2(1-12\,l^2+72\,l^4+(\frac{12\,\phi^*H^*}{(1-\beta)})+h^2(1-12\,l^2+(\frac{12\,\phi^*H^*}{(1-\beta)}))$$

$$+h(1-12\,l^2+72\,l^4+(\frac{12\,\phi^*H^*}{(1-\beta)}))\tanh(\frac{1}{2\,l})^3)$$

Appendix (B)

The output result of equation (23)

$$C_f = \frac{-1280((-1+\varepsilon)^3 - 12(-1+\varepsilon)\,l^2 - (\frac{12\,\phi^* H^*}{(1-\beta)}) - 24\,l^3\,\tanh(\frac{1-\varepsilon}{2\,l})}{\pi^3(1280 - 32\,\pi^2 + \pi^4)}$$

$$[\ \frac{1}{1-\varepsilon} - \frac{\varepsilon}{6(-1+\varepsilon)^2} - \frac{3}{(-1+\varepsilon)^3 - 12(-1+\varepsilon)\,l^2 - (\frac{12\,\phi^* H^*}{(1-\beta)}) - 24\,l^3\,\tanh(\frac{1-\varepsilon}{2\,l})}$$

$$\frac{3\,\varepsilon}{(-1+\varepsilon)^3 - 12(-1+\varepsilon)\,l^2 - (\frac{12\,\phi^* H^*}{(1-\beta)}) - 24\,l^3\,\tanh(\frac{1-\varepsilon}{2\,l})}\ ]$$

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