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Methodology Article

Research on Ratio-Extremum Algorithm for Topology Optimization of Continuum Structures Including Active Constraint Identification

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Abstract: To obtain the topology optimization algorithm of continuum structure which can effectively identify the effective constraints and quickly converge, based on the original Ratio-Extremum algorithm theory based on truss structure optimization, the emitter algorithm theory is introduced into the topology optimization of continuum structure. Firstly, taking pseudo density as design variables, mathematical model of the minimization mass with constraints of nodal displacements and element stresses is constructed. Secondly, according to essential extremum conditions of Dual objective function, iterative optimization direction and analytical step-size of constraint multipliers are derived. And, according to essential extremum conditions of Generalized Lagrange function, iterative optimization direction and analytical step-size of pseudo densities are derived. Analytical step-sizes are used to avoid one-dimensional optimization and then the calculation quantity of iterative optimization can be decreased. Thirdly, first-order partial derivatives of nodal displacement and element equivalent stress constraints with respect to pseudo densities are given. After that, by using self-compiled MATLAB program for continuum structure analysis, partial derivative calculation and optimization iteration, 4 optimization examples of different beam structures are used to show the changes of active nodal displacement and element equivalent stress constraints, and structural mass in the optimization iteration process, and to show the effectiveness of Ratio-Extremum algorithm in topology optimization of continuum structures.

Keywords: Continuum Structure, Topology Optimization, Ratio-Extremum Algorithm, Optimization Direction, Step-size, Active Constraint Identification

1. Introduction

Topology optimization is a very effective method to design structural shapes under specified loads, and performance constraints and boundary conditions, and to achieve better performance index [1]. In which, variable-density method [2] was earlier proposed in topology optimization, and has been continuously improved and developed with much more efforts of many researchers [3].

In early development of variable-density method, the continuum shape optimization was discussed by Bendsøe [4]. With that, the material interpolation schemes were used to solve the topology optimization of composite material [5]. And then, the process of variable-density topology optimization could be visualized by mathematical tools [6]. The visualization technique provides a good idea for engineering design.

In subsequent studies, some shortcomings of variable-density method have been greatly improved. The stability problem was introduced into traditional variable-density method with adding stability constraints to solve the problem of topology optimization by ZHAO [7] and ZHANG [8]. Smooth boundaries for irregular continuum structures were discussed by YU [9], LI [10] and LIU [11]. The boundary grids were used to fit a smooth curve or smooth surface at engineering practice. Weighted Pseudo densities were accelerated to approximate the 0/1 value in the optimization process by XU [12], DU [13] and YAN [14]. Thus, gray elements on the boundaries could be effectively simulated. And, the stability of topology optimization process and solving efficiency were improved. To achieve clear optimization boundaries without checkerboard phenomenon and mesh dependence, density gradient weighted function was used to automatically discriminate and weaken the filtering average effect of optimization boundary by LONG [15], LI [16] and ZHANG [17]. The model was post processed after topology optimization, and gray elements were filtered or suppressed to improve the solving efficiency by GAO [18], ZHANG [19] and DU [20]. Local quadratic programming on the basis of overall topology optimization was carried out by CHEN [21], LUO [22] and Namhee [23]. Isolated elements were deleted, and checkerboard elements were reformed to be more suitable to the engineering needs. A design space adjustment method without affecting the convergence of algorithm was proposed to solve the problems, such as large computation and material distribution in topology optimization, by YI [24] and ZHANG [25]. A multi-density method based on the homogenization method was discussed to realize the optimization design of variable-density lattice structures by LI [26] and LIAO [27].

Variable-density method was mainly improved with filtering algorithm or local optimization algorithm in post-processing. Total number of cycles was really reduced. Whereas, the calculation quantity of single cycle was increased. In the basis of optimization algorithm, the improvement on combining "optimization direction" and "step size" is still insufficient.

Ratio-extremum method is a new optimization algorithm from truss structures, which designates the optimization direction and step-size according to essential extremum conditions of Lagrange function and Dual function [28]. In which, the step-size can be determined by analytic method [29], rather than one-dimension searching. And, it can be also used to solve the optimization problem with fundamental frequency constraints [30]. The effectiveness is verified by optimization examples of truss structures.

Based on structural similarity, Ratio-extremum theory is to firstly extend to topology optimization of continuum structures, with pseudo-densities taken as design variables. Considering the problem of structural mass minimization with the constraints of nodal 1 displacements and element equivalent stresses, the optimization direction and step-size will be discussed by essential extremum conditions of Lagrange and Dual functions. The key points to realize this algorithm are first-order partial derivatives of nodal displacement and element equivalent stress constraints. And, the derivatives will be present. Then, by using self-compiled MATLAB program of structural analysis, derivative calculation and optimization iteration, the effectiveness is to be showed by optimization examples of four different beam structures.

2. Topology Optimization Model

Taking pseudo-density x as design variable, under the constraints of nodal displacements and stresses, and upper and lower limits of design variables, the continuum topology optimization problem of mass minimization can be expressed as:

$$\left.\begin{array}{ll}
Find & \boldsymbol{x}=[x_1,x_2,...,x_n,...,x_N]^T \in R^N \\
\min & M(\boldsymbol{x})=\sum_{n=1}^{N} x_n \rho_n v_n \\
s.t. & g_{ui}(\boldsymbol{x})=(\dfrac{u_i}{[u_i]}-1)\le 0 \quad (i=1,2,...,I) \\
 & g_{\sigma n}(\boldsymbol{x})=(\dfrac{\sigma_n}{[\sigma_n]}-1)\le 0 \quad (n=1,2,...,N) \\
 & g_{xk}(\boldsymbol{x})=-x_k\le 0 \quad (k=1,2,...,N) \\
 & g_{xj}(\boldsymbol{x})=x_j-1\le 0 \quad (n=1,2,...,N)
\end{array}\right\} \tag{1}$$

Wherein, $x=[x_1, x_2,..., x_n, \ldots, x_N]^T$ constitutes the vector of design variables, and superscript T is the symbol of vector transposition. R^N represents N-dimensional real number space. $M(x)$ expresses the objective function of minimizing structural mass. ρ_n is physical density of n-th element. v_n is the volume. $g_{ui}(x)$ represents the displacement constraint of i-th node, and u_i is the displacement component, and $[u_i]$ is the allowable value, and I is total number of nodal displacement constraints. $g_{\sigma n}(x)$ represents the stress constraint of n-th element, and σ_n is the element stress, and $[\sigma_n]$ is the allowable value, and N is total number of elements. $g_{xk}(x)$ is set as lower limit constraint of k-th design variable, and $g_{xj}(x)$ is set as upper limit constraint of j-th design variable.

3. Topology Optimization Algorithm

3.1. Optimization Principle

From the objective function of Equation (1) and the constraint functions of nodal displacements, element stresses and design variables, Generalized Lagrange function can be expressed as:

$$\begin{aligned}
L(\boldsymbol{x},\lambda)=M(\boldsymbol{x})+\sum_{i=1}^{I}\lambda_{ui}g_{ui}(\boldsymbol{x})+\sum_{n=1}^{N}\lambda_{\sigma n}g_{\sigma n}(\boldsymbol{x})+ \\
\sum_{k=1}^{N}\lambda_{xk}g_{xk}(\boldsymbol{x})+\sum_{j=1}^{N}\lambda_{xj}g_{xj}(\boldsymbol{x})
\end{aligned} \tag{2}$$

Wherein, λ_{ui} is the constraint multiplier corresponding to the displacement of i-th node, and $\lambda_{\sigma n}$ is the constraint multiplier corresponding to the stress of n-th element, and λ_{xk} is the constraint multiplier corresponding to lower limit of k-th design variable, and λ_{xj} is the constraint multiplier corresponding to upper limit of j-th design variable, and λ represents the vector of constraint multipliers. The essential extremum condition of Equation (2) is:

$$\nabla L(\boldsymbol{x}^*, \boldsymbol{\lambda}^*) = 0 \tag{3}$$

Dual programming problem of Equation (1) can be expressed as:

$$\left.\begin{array}{ll} \text{Find} & \boldsymbol{\lambda} \in R^{3N+I} \\ \max & \phi(\boldsymbol{\lambda}) = \min\limits_{x}\left[L(\boldsymbol{x},\boldsymbol{\lambda})\right] = L\left[\boldsymbol{x}^*(\boldsymbol{\lambda}),\boldsymbol{\lambda}\right] \\ \text{s.t.} & \boldsymbol{\lambda} \geq 0 \end{array}\right\} \tag{4}$$

The essential extremum condition of Equation (4) is:

$$\frac{\partial L}{\partial \mathrm{x}}\frac{\partial \mathrm{x}}{\partial \lambda} + \frac{\partial L}{\partial \lambda} = 0 \tag{5}$$

That means, at optimal point x^*, Equation (3) holds and active constraints are on the critical, the corresponding multipliers are non-negative. And, inactive constraints are within their bounds, the corresponding multipliers should be zero.

3.2. Algorithm Idea of Ratio-extremum

Firstly, a searching optimization is implemented in multiplier space. That is, the searching direction of constraint multipliers is determined by the second term at left end of Equation (5), and its step-size can be determined by Equation (5). If the multipliers calculated is less than or equal to zero, just make them being zero. Secondly, another searching optimization is implemented in design-variable space. That is, the searching direction of design variables is determined by Equation (3), and its step-size can be determined by partial derivative of Generalized Lagrange function with respect to the step-size.

Repeat the above two steps to convergence. If the values of design variables are close to the lower bounds, the corresponding elements can be deleted to obtain the topology optimization result.

3.3. Iteration of Constraint Multipliers

The iterative solution of constraint multipliers can be expressed as:

$$\lambda^{(k+1)} = \lambda^{(k)} - \beta^{(k)} \mathrm{d}_{\lambda}{}^{(k)} \tag{6}$$

Wherein, d_λ represents the searching direction of constraint multipliers. And, β is the step-size. Superscript (k) indicates that the variable is the quantity of k-th step.

By the second term at left end of Equation (5), the searching direction of constraint multipliers can be determined, and the expression is as follows:

$$\mathrm{d}_{\lambda} = \begin{bmatrix} -g_1 & -g_2 & \cdots & -g_l & \cdots & -g_L \end{bmatrix}^{\mathrm{T}} \tag{7}$$

Wherein, g_l represents the l-th constraint. And L is total number of constraints.

The step-size of constraint multipliers can be determined by Equation (5), that is:

$$\beta = \frac{2\mathrm{d}_{\lambda}{}^{\mathrm{T}}\mathrm{d}_{\lambda}}{\mathrm{d}_{\lambda}{}^{\mathrm{T}}\mathrm{G}_{\mathrm{x}}\mathrm{G}_{\lambda}\mathrm{d}_{\lambda}} \tag{8}$$

Wherein, G_x represents Jacobian matrix of constraints with respect to design variables. G expresses Jacobian matrix of objective function and constraints with respect to design variables [29].

In the iterative process of Equation (6), if one of constraint multipliers is positive, it indicates that the constraint is an active constraint. And, if one of constraint multipliers is negative, it means that the constraint does not work temporarily, and its multiplier should be zero.

3.4. Iteration of Pseudo-densities

The iterative solution of pseudo-densities can be expressed as:

$$\mathrm{x}^{(k+1)} = \mathrm{x}^{(k)} - \alpha^{(k)} \mathrm{d}_{\mathrm{x}}{}^{(k)} \tag{9}$$

Wherein, d_x represents the searching direction of pseudo-densities. And, α is the step-size.

Multiply the square of pseudo density x_m (m=1, 2, …, N) by Equation (3). And then, apply the solution formula of quadratic equation to determine the searching direction as follows:

$$d_{x_m} = x_m - x_m \sqrt{\frac{-\left(\sum\limits_{i=1}^{I}\lambda_{ui}\frac{\partial g_{ui}}{\partial x_m} + \sum\limits_{n=1}^{N}\lambda_{\sigma n}\frac{\partial g_{\sigma n}}{\partial x_m} + \sum\limits_{k=1}^{N}\lambda_{xk}\frac{\partial g_{xk}}{\partial x_m} + \sum\limits_{j=1}^{N}\lambda_{xj}\frac{\partial g_{xj}}{\partial x_m}\right)}{\rho_m v_m}} \tag{10}$$

Wherein,

$$\frac{\partial g_{ui}}{\partial x_m} = -\frac{\mathrm{e}_i}{\left[u_i\right]}\mathrm{K}^{-1}\frac{\partial \mathrm{K}}{\partial x_m}\mathrm{K}^{-1}\mathrm{F} \tag{11}$$

$$\frac{\partial g_{\sigma n}}{\partial x_m} = \frac{1}{\left[\sigma_{neqv}\right]}\frac{\partial \sigma_{neqv}}{\partial x_m} \tag{12}$$

$$\frac{\partial \sigma_{\mathrm{j}}}{\partial x_m} = \mathrm{S}\frac{\partial \mathrm{u}_{\mathrm{j}}}{\partial x_m} \tag{13}$$

$$\sigma_{eqv} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \tag{14}$$

Among them, K expresses overall stiffness matrix. F is nodal load vector. And, e_i represents the row vector in which the i-th component is 1 and the others are zero. And, σ_{neqv} is equivalent stress of n-th element. S is the stress matrix.

If the value in the square root of Equation (10) is negative, the pseudo-density can be set as its lower limit. So, the searching direction should be:

$$d_{xn} = x_n \tag{15}$$

By applying first-order partial derivative of Generalized Lagrange function with respect to the step-size of pseudo-densities, Equation (9) can be used to determine the step-size as:

$$\alpha = \frac{\left(\nabla \mathrm{L}_x\right)^{\mathrm{T}} \mathrm{d}_x}{\left|\nabla \mathrm{L}_x\right| \cdot \left|\mathrm{d}_x\right|} \tag{16}$$

If one of pseudo-densities exceeds its lower limit, just set the corresponding term of the vectors to zero.

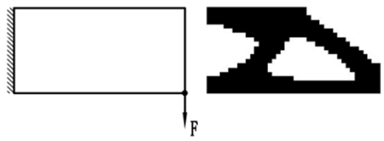
4. Numerical Examples

The programs of continuum structure analysis, partial derivative calculation and optimization iteration are compiled by MATLAB software, and the optimization results are presented by the grid graphs of black (1) and white (0).

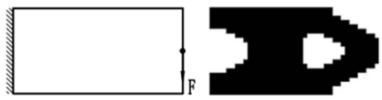
The basic conditions of four examples are as follows: one of continuum structures is divided into 40×20 elements, the length and width of each element are 0.4 unit length, the thickness is 0.001 unit length, and the density is 78×10^5 unit density. The nodal displacements are less than or equal to 0.003 unit length, and the element equivalent stresses are less than or equal to 126×10^6 unit stress. The iterative termination condition is set as relative change rate of pseudo-densities is less than or equal to 1×10^{-7}. The following figures only show the case that the iteration number is up to 200.

The fixed nodes and loadings of four different beam structures are shown in Figure 1. In which, F is 10000 unit force, and F_1, F_2 and F_3 are 3500 unit forces, respectively. The left figures of (a), (b), (c) and (d) in Figure 1 are the schematics of beam structures, and the right figures are the results of topology optimization.

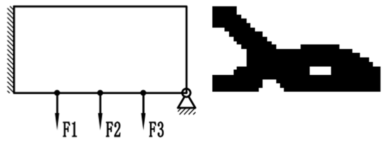
In order to verify whether the results meet the constraint and convergent conditions, Figure 2, Figure 3 and Figure 4 show the changes of the maximum nodal displacements and element equivalent stresses that the performance constraints are active at final, as well as the masses of four different beam structures, respectively. In Figure 2 and Figure 3, the nodal displacements and element equivalent stresses being active at final present great changes in the iteration process. The phenomenon is caused by the active constraints switching to inactive ones. This indicates that the beam structures are unstable. When the curves change gently, the switching phenomenon does not appear, and the beam structures are stable. This means that the iteration of Equation (6) for constraint multipliers can effectively identify active constraints.



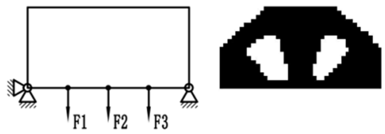
(a) Cantilever beam of fixing left end and loading right corner



(b) Cantilever beam of fixing left end and loading right center



(c) Beam of fixing left end and supporting lower right corner with 3 loadings



(d) Simply supported beam with 3 loadings

Figure 1. *Beam structures and optimization results.*

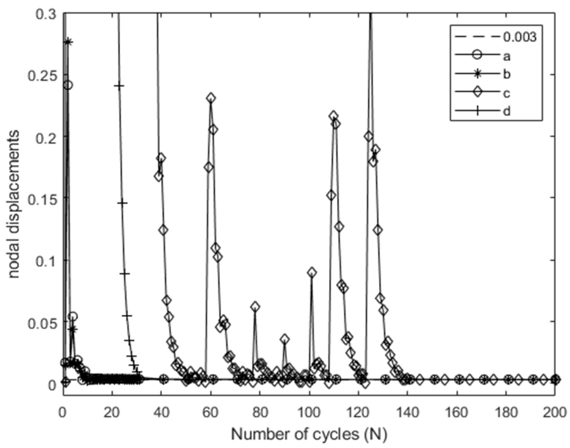


Figure 2. *Active nodal displacements.*

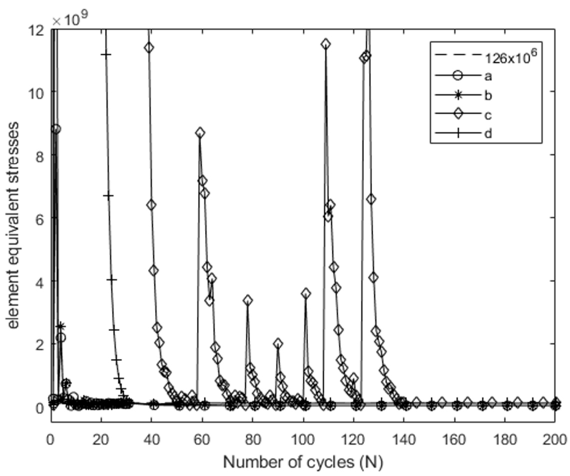


Figure 3. *Active element equivalent stresses.*

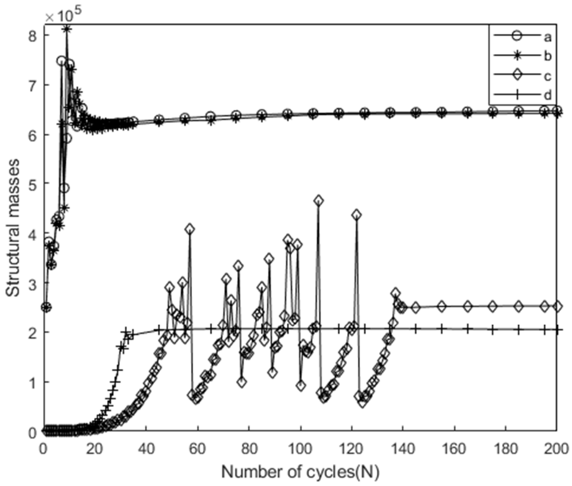


Figure 4. *Structural masses.*

By analyzing the iterative results, it presents that the maximum nodal displacements of four different beam structures converge to 0.003, and the maximum element equivalent stresses are all within 126×10^6, which satisfy the conditions of displacement and stress constraints.

After 40 iterations, the maximum nodal displacements and element equivalent stresses, and structural masses of beam structures (a), (b) and (d) in Figure 1 can tend to be stable. However, the ones of beam structure (c) just tend to be stable after 140 iterations. And, the objective function of mass can smoothly transition to convergence. This means that the iteration of Equation (9) for pseudo densities can converge quickly when the active constraints do not change.

5. Conclusion

According to the theory of Ratio-Extremum method for structural optimization and essential extremum conditions of optimization problem, this paper mainly discusses the searching direction and step-size of constraint multipliers and pseudo-densities by analytical method for continuum structures. Based on matrix displacement method of continuum structure analysis, the first-order partial derivatives of nodal displacement and element equivalent stress constraints with respect to pseudo-densities are given.

By using self-written MATLAB for continuum structure analysis, partial derivative calculation and optimization iteration, the optimization examples of 4 different beam structures are used to show: the iteration of constraint multipliers can effectively identify whether the constraints are active or not. When the iteration converges, the active constraints reach to critical. The iteration of pseudo-densities can converge quickly when the active constraints do not change.

After that, based on the existing research results, the following two research points will be carried out: 1. Further refine the structural grid to make the optimized structure smoother; 2. Extend the algorithm to 3D structure optimization to make it more in line with the Practical engineering application. Mesh refinement and three-dimensional structure lead to the increase of computing units, which means the increase of computing cost and overcoming the problem of small computer running memory.

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