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A New DEA Cross-Efficiency Method Based on Consensus

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Abstract: Data Envelopment Analysis (DEA), as a non-parametric technique for evaluating the relative efficiencies of a set of homogenous decision-making units (DMUs) with multi-inputs and multi-outputs, is widely used in the performance evaluation field. This paper develops a new DEA cross-efficiency method based on consensus (CEC-DEA), which can be more reasonably and effectively in evaluating and ranking decision-making units (DMUs). In our proposed method, we firstly attempt to obtain the unique set of weights for each DMU through a second-objective model which can minimize the total variance between the self-evaluated efficiencies and the peer-evaluated efficiencies. Then, based on the acquired weights, the cross-efficiency scores of all DMUs are calculated. Before the cross-efficiency aggregation, we choose a DMU as the Common Reference Point (CRP) based on which all the cross-efficiency scores are rescaled to be comparable for the aggregation. In the aggregation stage, we define the Evaluation Consensus Degrees (ECDs) of all DMUs, which are used as the weights to aggregate the cross-efficiency scores through the weighted geometric mean method. Comprehensively, the proposed CEC-DEA method is developed to increase the rationality and acceptability of the evaluations for all DMUs. Finally, the a numerical example is illustrated to prove the effectiveness of the proposed method.

Keywords: DEA, Cross-Efficiency, Common Reference Point, Aggregation, Evaluation Consensus Degree

1. Introduction

Data Envelopment Analysis (DEA), developed by Charnes et al., is a non-parametric technique for evaluating the relative efficiencies of a set of homogenous decision-making units (DMUs) with multi-inputs and multi-outputs [1]. Because of its effectiveness in identifying the best-practice frontier and ranking DMUs, DEA has been widely applied in the performance evaluation in various fields [2-6]. However, the traditional DEA model acquires the efficiency results in a self-evaluation mode, which allows each DMU to measure its efficiency using its favorable weights. However, this self-interested characteristic decreases the fairness and equality of the efficiency evaluation among DMUs [7] and cannot differentiate the efficient DMUs [8]. Furthermore, the traditional DEA model is also criticized for its inability to ensure the uniqueness of the optimal weights for each DMU [9, 10].

To address the drawbacks caused by the self-evaluation mode of the traditional DEA model, Sexton et al. proposed the cross-efficiency evaluation method [11]. Incorporating self-evaluation and peer-evaluation, the cross-efficiency evaluation method reflects the feature of group evaluation. Its advantages of the cross-efficiency evaluation are obvious. For example, the cross-efficiency evaluation method can effectively distinguish good and poor performers and produce a full ranking of DMUs [12]. In addition, it can also avoid the issue of unrealistic weight schemes without incorporating weight restrictions [13, 14].

Despite its advantages, the cross-efficiency evaluation method still fails to ensure the uniqueness of the optimal weights generated for each DMU [15]. To resolve this problem, many researchers have proposed modified cross-efficiency models by introducing a second-objective function. For example, Doyle and Green proposed the aggressive and benevolent models (the ACE method and the NCE method) to acquire the optimal set of weights for cross-efficiency evaluations [15]. Liang et al. suggested three second-objective formulations based on variations from the expected values [16]. Wang and Chin developed a neutral cross-efficiency model (NCE) with a secondary goal that a

DMU searches for a set of input and output weights to maximize its efficiency as a whole and at the same time to make its each output being as efficient as possible to produce sufficient efficiency as an individual [17]. Meanwhile, [17] also gives another cross-weight evaluation (CWE) way for assessing and ranking DMUs. By introducing a virtual ideal DMU (IDMU) and a virtual anti-ideal DMU (ADMU), Wang et al. determined the weights based on ideal and anti-ideal decision making units without the need to be aggressive or benevolent toward any DMUs [18]. Jahanshahloo et al. proposed the symmetric weight assignment technique (SWAT) to perform a symmetric selection of weights [19]. Contreras introduced the cross-efficiency model with a second goal of minimizing the ranking orders of the front DMUs [20]. Considering both the desirable and undesirable cross-efficiency targets of all DMUs, Wu et al. developed several secondary-goal models to obtain unique weights [21].

The cross-efficiency aggregation method is a critical component of the cross-efficiency evaluation method, and it has also drawn the attention of several researchers [22]. For example, Wu et al. applied Shannon entropy to the cross-efficiency aggregation method [22, 23]. Wang et al. adopted the ordered weighted averaging (OWA) operator for cross-efficiency aggregation [24]. Yang et al. proposed the evidential-reasoning (ER) approach for aggregating cross-efficiencies [25]. Wang et al. developed approaches to determine the relative importance weights for aggregating the cross-efficiency scores [26].

The cross-efficiency method incorporates self-evaluation and peer-evaluation; thus, it can be considered as a group evaluation method. As noted in [27], group evaluation methods should favor decision-making techniques designed to garner wide consensus among all evaluators. Classically, consensus is defined as the full and unanimous agreement of all decision makers regarding all the possible alternatives [28, 29]. To achieve more acceptable and reasonable evaluation results, it is a possible way to introduce the concept of "consensus" into the cross-efficiency method [30].

In this paper, we propose a DEA cross-efficiency method based on consensus (CEC-DEA). Specifically, we define the Evaluation Consensus Degree (ECD) of each DMU and aggregate the cross-efficiency scores using the weighted geometric mean method, with the ECDs used as the weights. To the best of our knowledge, the use of ECDs as the weights in the cross-efficiency aggregation has not been applied in previous research. Moreover, we apply the geometric mean method because the efficiency scores are ratio figures and the geometric mean method performs better than the arithmetic mean method when calculating the mean values of ratio figures [32, 33].

As another important component of our CEC-DEA method, a second-objective model is constructed to ensure the uniqueness of the weights generated for each DMU. In particular, we also base our second-objective model on the concept of consensus, which minimizes the total variance between the peer-evaluated efficiencies and the self-evaluated efficiencies.

Furthermore, because the cross-efficiency scores of each DMU are calculated under the peer -evaluation of different DMUs (i.e., they are obtained with different sets of weights), these scores essentially cannot be compared; therefore, they are unsuitable for direct aggregation [7-9, 19]. To ensure that the cross-efficiency scores are comparable, we choose a DMU as the Common Reference Point (CRP) and rescale all cross-efficiency scores based on this DMU before aggregation.

The remainder of the paper is organized as follows. In section 2, we present the previous DEA models, including the traditional CCR-DEA model and the cross-efficiency models. In section 3, we propose our CEC-DEA method. In section 4, the proposed CEC-DEA method is illustrated through a numerical example. Finally, in section 5, we present our conclusions.

2. Traditional CCR-DEA Model and Cross-Efficiency Evaluation Methods

2.1. Traditional CCR-DEA Model

Assume that there are n comparable DMUs to be evaluated and that each $\mathrm{DMU}_k\left(k=1,2,\cdots,n\right)$ has m input(s) and t output(s), which are denoted as $x_{ik}\left(i=1,2,\cdots,m\right)$ and $y_{rk}\left(r=1,2,\cdots,t\right)$, respectively. For $\mathrm{DMU}_k\left(k=1,2,\cdots,n\right)$, consider that v_{ik} and u_{rk} are the weights of the ith input and the rth output, respectively. The traditional CCR-DEA model proposed by Charnes et al. [1] is shown in model (1).

$$
\begin{aligned}
&\max\ \theta_k=\frac{\sum_{r=1}^{t}u_{rk}y_{rk}}{\sum_{i=1}^{m}v_{ik}x_{ik}}\\
&\text{s.t.}\quad \theta_{jk}=\frac{\sum_{r=1}^{t}u_{rk}y_{rj}}{\sum_{i=1}^{m}v_{ik}x_{ij}}\le 1,\quad j=1,2,\cdots,n,\\
&\qquad u_{rk}\ge 0,\quad r=1,2,\cdots,t,\\
&\qquad v_{ik}\ge 0,\quad i=1,2,\cdots,m.
\end{aligned}
\tag{1}
$$

Through linear transformation, model (1) can be converted to the following linear programming model (2):

$$
\begin{aligned}
&\max\quad \theta_k=\sum_{r=1}^{t}u_{rk}y_{rk}\\
&\text{s.t.}\quad \sum_{i=1}^{m}v_{ik}x_{ik}=1,\\
&\qquad \sum_{r=1}^{t}u_{rk}y_{rj}-\sum_{i=1}^{m}v_{ik}x_{ij}\le 0,\quad j=1,2,\cdots,n,\\
&\qquad u_{rk}\ge 0,\qquad r=1,2,\cdots,t,\\
&\qquad v_{ik}\ge 0,\qquad i=1,2,\cdots,m.
\end{aligned}
\tag{2}
$$

In models (1) and (2), θ_k denotes the efficiency of $\text{DMU}_k\ (k=1,2,\cdots,n)$ based on self-evaluations. Based on model (2), we can obtain the optimal weights $v_{ik}\ (i=1,2,\cdots,m)$ and $u_{rk}\ (r=1,2,\cdots,t)$ for $\text{DMU}_k\ (k=1,2,\cdots,n)$.

2.2. Cross-Efficiency Evaluation Methods

According to the cross-efficiency method proposed by Sexton et al. (1986), the cross-efficiency value $\theta_{jk}\ (j=1,2,\cdots,n;k=1,2,\cdots,n)$ of DMU_j under the evaluation of $\text{DMU}_k\ (k=1,2,\cdots,n)$ is calculated as follows:

$$\theta_{jk}=\frac{\sum_{r=1}^{t}u_{rk}y_{rj}}{\sum_{i=1}^{m}v_{ik}x_{ij}},\quad j=1,2,\cdots,n;k=1,2,\cdots,n. \tag{3}$$

According to (3), the cross-efficiency scores of all DMUs are obtained. Then, based on (4), the final efficiency A_j of DMU_j is obtained.

$$A_j=\frac{\sum_{k=1}^{n}\theta_{jk}^{r}}{n},\quad j=1,2,\cdots,n. \tag{4}$$

However, the cross-efficiency method still fails to ensure the uniqueness of weights, which are obtained from model (2). To solve this problem, Doyle and Green [14] developed the benevolent and aggressive models, which are shown as models (5) and (6), respectively.

$$\begin{aligned}
\max\ & \sum_{r=1}^{t}u_{rk}\Big(\sum_{j=1,j\neq k}^{n}y_{rj}\Big)\\
\text{s.t.}\ & \sum_{i=1}^{m}v_{ik}\Big(\sum_{j=1,j\neq k}^{n}x_{ij}\Big)=1,\\
& \sum_{r=1}^{t}u_{rk}y_{rk}-\theta_k\sum_{i=1}^{m}v_{ik}x_{ik}=0,\\
& \sum_{r=1}^{t}u_{rk}y_{rj}-\sum_{i=1}^{m}v_{ik}x_{ij}\le 0,\quad j=1,2,\cdots,n,\\
& u_{rk}\ge 0,\quad r=1,2,\cdots,t,\\
& v_{ik}\ge 0,\quad i=1,2,\cdots,m.
\end{aligned} \tag{5}$$

$$\begin{aligned}
\min\ & \sum_{r=1}^{t}u_{rk}\Big(\sum_{j=1,j\neq k}^{n}y_{rj}\Big)\\
\text{s.t.}\ & \sum_{i=1}^{m}v_{ik}\Big(\sum_{j=1,j\neq k}^{n}x_{ij}\Big)=1,\\
& \sum_{r=1}^{t}u_{rk}y_{rk}-\theta_k\sum_{i=1}^{m}v_{ik}x_{ik}=0,\\
& \sum_{r=1}^{t}u_{rk}y_{rj}-\sum_{i=1}^{m}v_{ik}x_{ij}\le 0,\quad j=1,2,\cdots,n,\\
& u_{rk}\ge 0,\quad r=1,2,\cdots,t,\\
& v_{ik}\ge 0,\quad i=1,2,\cdots,m.
\end{aligned} \tag{6}$$

In models (5) and (6), θ_k is the CCR efficiency of DMU_k obtained from model (2). As shown in these models, the benevolent model (5) aims to maximize the cross-efficiencies of the other DMUs, whereas the aggressive model (6) seeks to minimize the cross-efficiencies of the other DMUs.

3. Proposed CEC-DEA Method

In our CEC-DEA method, we first attempt to obtain the unique set of weights for each DMU through a second-objective model that minimizes the total variance between the self-evaluated efficiencies and the peer-evaluated efficiencies. Then, based on the acquired weights, the cross-efficiency scores of all DMUs are calculated. Before the cross-efficiency aggregation, we choose a DMU as the *CRP* and rescale all the cross-efficiency scores so that they are comparable for the aggregation. In the aggregation stage, we define the *ECD* of each DMU and aggregate the cross-efficiency scores through the weighted geometric mean method, with the *ECD*s used as the weights. Comprehensively, the proposed CEC-DEA method is developed to increase the reasonability and acceptability of the evaluations for all DMUs.

3.1. Second-Objective Model

In the first stage of the CEC-DEA method, we construct the second-objective model to obtain the unique set of weights for each DMU. The second-objective model is constructed based on the concept of consensus, which is shown in model (7). Specifically, under the peer-evaluation of each DMU_k, the total variance between the cross-efficiency scores $\theta_{jk}\ (j=1,2,\cdots,n)$ and the self-evaluated efficiencies $\theta_j^{c}\ (j=1,2,\cdots,n)$ of all DMUs calculated from the traditional CCR model is minimized.

$$\min \sum_{j=1}^{n}(\theta_j^c - \theta_{jk})$$
$$\text{s.t.} \quad \sum_{i=1}^{m} v_{ik} x_{ik} = 1,$$
$$\theta_{jk} = \frac{\sum_{r=1}^{t} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1, \quad j = 1,2,\cdots,n, \tag{7}$$
$$\theta_{kk} = \theta_k^c,$$
$$u_{rk} \ge 0, \quad r = 1,2,\cdots,t,$$
$$v_{ik} \ge 0, \quad i = 1,2,\cdots,m.$$

In model (7), θ_j^c and θ_k^c are the self-evaluated efficiencies of DMU_j and DMU_k, respectively, which were calculated from the traditional CCR model; and θ_{jk} is denoted as the cross-efficiency score of DMU_j under the peer-evaluation of DMU_k, which is calculated according to (3).

Because the self-evaluated efficiency θ_j^c of $\mathrm{DMU}_j \left(j = 1,2,\cdots,n\right)$ is a fixed value obtained from the CCR model, the objective function in model (7) is equivalent to maximizing the sum of the cross-efficiency of DMU_j, i.e., $\mathrm{Max}\sum_{j=1}^{n}\theta_{jk}$. As suggested by Sexton et al. (1986), $\sum_{j=1}^{n}\theta_{jk}$ can be transformed as follows:

$$\sum_{j=1}^{n}\theta_{jk} = \sum_{j=1}^{n}(\sum_{r=1}^{t}(u_{rk}y_{rj} - \sum_{i=1}^{m}(v_{ik}x_{ij}) = \sum_{r=1}^{t}(u_{rk}\sum_{j=1}^{n}y_{rj})\text{-}\sum_{i=1}^{m}(v_{ik}\sum_{j=1}^{n}x_{ij}) \tag{8}$$

Therefore, model (7) can be transformed into model (9):

$$\max \sum_{r=1}^{t}(u_{rk}\sum_{j=1}^{n}y_{rj})\text{-}\sum_{i=1}^{m}(v_{ik}\sum_{j=1}^{n}x_{ij})$$
$$\text{s.t.} \quad \sum_{i=1}^{m} v_{ik} x_{ik} = 1,$$
$$\sum_{r=1}^{t} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \quad j = 1,2,\cdots,n, \tag{9}$$
$$\sum_{r=1}^{t} u_{rk} y_{rk} - \theta_k^c \sum_{i=1}^{m} v_{ik} x_{ik} = 0,$$
$$u_{rk} \ge 0, \quad r = 1,2,\cdots,t,$$
$$v_{ik} \ge 0, \quad i = 1,2,\cdots,m.$$

By solving models (2) and (9), a unique set of weights $v_{ik}\left(i = 1,2,\cdots,m\right)$ and $u_{rk}\left(r = 1,2,\cdots,t\right)$ can be calculated for each $\mathrm{DMU}_k \left(k = 1,2,\cdots,n\right)$.

3.2. Rescale the Cross-Efficiency Scores

As mentioned in the Introduction, the cross-efficiency scores cannot be directly aggregated because they are obtained with using different sets of weights [7-9]. To ensure that the cross-efficiency scores are comparable for aggregation, we attempt to choose a DMU as the *CRP* and rescale all the cross-efficiency scores based on this DMU. Specifically, if DMU_r is chosen as the *CRP*, then the cross-efficiency score $\theta_{jk}\left(j = 1,2,\cdots,n; k = 1,2,\cdots,n\right)$ of $\mathrm{DMU}_j \left(j = 1,2,\cdots,n\right)$ under the peer-evaluation of $\mathrm{DMU}_k \left(k = 1,2,\cdots,n\right)$ can be rescaled as θ_{jk}^r, which is shown as follows:

$$\theta_{jk}^r = \frac{\theta_{jk}}{\theta_{rk}}, \quad j = 1,2,\cdots,n; k = 1,2,\cdots,n. \tag{10}$$

In (10), $\theta_{rk}\left(k = 1,2,\cdots,n\right)$ denotes the cross-efficiency score of DMU_r under the peer-evaluation of $\mathrm{DMU}_k \left(k = 1,2,\cdots,n\right)$. After rescaling, the new cross-efficiency scores $\theta_{jk}^r \left(j = 1,2,\cdots,n; k = 1,2,\cdots,n\right)$ are comparable and thus suitable for aggregation. To ensure that the new cross-efficiency scores $\theta_{jk}^r \left(j = 1,2,\cdots,n; k = 1,2,\cdots,n\right)$ are less than or equal to 1.0 as much as possible, the selected DMU as the *CRP* had better be the one with the cross-efficiency scores of 1.0 for most cases.

3.3. Cross-Efficiency Scores Aggregation

To achieve more acceptable and reasonable evaluation results, the aggregation stage of our CEC-DEA method considers the consensus among all DMUs. Classically, consensus is defined as the full and unanimous agreement of all decision makers regarding all possible alternatives [29, 31]. Here, we define the *ECD* as follows:

$$c_k = \frac{1}{1 + \sqrt{\sum_{j=1}^{n}(\theta_{jk}^r - \overline{\theta_j})^2}}, \quad k = 1,2,\cdots,n. \tag{11}$$

In (11), $c_k\left(k = 1,2,\cdots,n\right)$ is denoted as the *ECD* of $\mathrm{DMU}_k \left(k = 1,2,\cdots,n\right)$ and $\overline{\theta_j}\left(j = 1,2,\cdots,n\right)$ is the geometric mean value of the rescaled cross-efficiency score $\theta_{jk}^r \left(j = 1,2,\cdots,n; k = 1,2,\cdots,n\right)$ of $\mathrm{DMU}_j \left(j = 1,2,\cdots,n\right)$, which is calculated as follows:

$$\overline{\theta_j} = \sqrt[n]{\prod_{k=1}^{n}\theta_{jk}^r}, \quad j = 1,2,\cdots,n. \tag{12}$$

Furthermore, $\sqrt{\sum_{j=1}^{n}(\theta_{jk}^{r}-\overline{\theta_j})^2}\ \left(k=1,2,\cdots,n\right)$ in (11) expresses the Euclidean distance between the vector θ_{jk}^{r} $\left(j=1,2,\cdots,n\right)$ and the vector $\overline{\theta_j}$ $\left(j=1,2,\cdots,n\right)$. From (11), the *ECD* of DMU_k, which is denoted as c_k, is shown to be inversely related to the calculated Euclidean distance $\sqrt{\sum_{j=1}^{n}(\theta_{jk}^{r}-\overline{\theta_j})^2}$. Because the minimum value of the Euclidean distance is zero, the *ECD*s belong to the variation range $(0,1]$. For example, when the Euclidean distance is zero, $c_k=1$ is obtained as the largest value.

Finally, we aggregate the cross-efficiency scores via the weighted geometric mean method, with the *ECD*s used as the weights. This geometric mean method is applied because the efficiency scores are ratio figures and the geometric mean method performs better than the arithmetic mean method when calculating the mean values of ratio figures [29, 30]. Therefore, the final efficiency score A_j^* of DMU_j is calculated as follows (13):

$$A_j^*=\sqrt[\sum_{k=1}^{n}c_k]{\prod_{k=1}^{n}\left(\theta_{jk}^{r}\right)^{c_k}},\quad j=1,2,\cdots,n. \tag{13}$$

To conclude, the steps of the proposed CEC-DEA method are outlined as follows:

Step 1: Calculate the weights for each DMU based on models (2) and (9);

Step 2: According to (3), calculate the cross-efficiency scores of all DMUs using the obtained weights;

Step 3: According to (10), rescale the cross-efficiency scores based on the selected *CRP*;

Step 4: According to (11), calculate the *ECD* of each DMU;

Step 5: According to (13), aggregate the rescaled cross-efficiency scores using the weighted geometric mean method with the *ECD*s as the weights;

Step 6: Rank all DMUs according to their final efficiency scores $A_j^*\left(j=1,2,\cdots,n\right)$.

4. Numerical Example

For the convenience of comparison, the proposed CEC-DEA method is illustrated using a previous numerical example from the literature [10]. As shown in Table 1, the numerical example includes fourteen DMUs, each of which has three inputs (i.e., Input1 = aircraft capacity in ton kilometers; Input2 = operating cost; and Input3 = non-flight assets and two outputs) and two outputs (i.e., Output1 = passenger kilometers; and Output2 = non-passenger revenue).

Table 1. *Example data.*

DMUs	Input1	Input2	Input3	Output1	Output2
DMU_1	5723	3239	2003	26677	697
DMU_2	5895	4225	4557	3081	539
DMU_3	24099	9560	6267	124055	1266
DMU_4	13565	7499	3213	64734	1563
DMU_5	5183	1880	783	23604	513
DMU_6	19080	8032	3272	95011	572
DMU_7	4603	3457	2360	22112	969
DMU_8	12097	6779	6474	52,363	2001
DMU_9	6587	3341	3581	26504	1297
DMU_{10}	5654	1878	1916	19277	972
DMU_{11}	12559	8098	3310	41925	3398
DMU_{12}	5728	2481	2254	27754	982
DMU_{13}	4715	1792	2485	31332	543
DMU_{14}	22793	9874	4145	122528	1404

4.1. Calculating the Efficiencies Based on the Proposed CEC-DEA Method

Based on model (2) and our second-objective model (9), the unique set of weights for each DMU is obtained. According to (3), we can obtain the cross-efficiencies $\theta_{jk}\left(j,k\in[1,14]\right)$ of all DMUs, which are shown in Table 2.

Table 2. *Cross-efficiency matrix.*

DMUj	DMUk													
	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10	DMU11	DMU12	DMU13	DMU14
DMU_1	0.86836	0.45013	0.62251	0.86836	0.84920	0.47261	0.81076	0.78813	0.70308	0.75116	0.86836	0.77130	0.84920	0.84920
DMU_2	0.17189	0.33794	0.04718	0.17189	0.17351	0.02471	0.24789	0.27242	0.28083	0.20577	0.17189	0.20249	0.17351	0.17351
DMU_3	0.88259	0.19416	0.94752	0.88259	0.88442	0.68979	0.72320	0.68331	0.62254	0.78461	0.88259	0.80717	0.88442	0.88442
DMU_4	0.95809	0.42586	0.70342	0.95809	0.94131	0.69730	0.82280	0.78498	0.69914	0.81125	0.95809	0.83407	0.94131	0.94131
DMU_5	0.96528	0.36582	1.00000	0.96528	1.00000	1.00000	0.77038	0.73590	0.77775	1.00000	0.96528	1.00000	1.00000	1.00000
DMU_6	0.88179	0.11080	0.95628	0.88179	0.87799	0.97658	0.66152	0.60837	0.50992	0.71759	0.88179	0.74779	0.87799	0.87799
DMU_7	0.92108	0.77806	0.47727	0.92108	0.87951	0.33816	1.00000	1.00000	0.83947	0.78077	0.92108	0.80117	0.87951	0.87951
DMU_8	0.78131	0.61137	0.51623	0.78131	0.77025	0.29237	0.84576	0.85876	0.82083	0.75318	0.78131	0.76308	0.77025	0.77025
DMU_9	0.78545	0.72775	0.50755	0.78545	0.78885	0.26768	0.87824	0.90719	0.94774	0.83745	0.78545	0.83692	0.78885	0.78885
DMU_{10}	0.78214	0.63539	0.65200	0.78214	0.82498	0.35640	0.77799	0.79436	1.00000	1.00000	0.78214	0.97192	0.82498	0.82498
DMU_{11}	1.00000	1.00000	0.42868	1.00000	1.00000	0.44179	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
DMU_{12}	0.94619	0.63364	0.75002	0.94619	0.96020	0.43951	0.93618	0.93951	0.99984	1.00000	0.94619	1.00000	0.96020	0.96020
DMU_{13}	1.00000	0.42565	1.00000	1.00000	1.00000	0.45551	1.00000	1.00000	1.00000	0.98429	1.00000	1.00000	1.00000	1.00000
DMU_{14}	1.00000	0.22767	1.00000	1.00000	1.00000	1.00000	0.77950	0.72754	0.64784	0.85685	1.00000	0.88375	1.00000	1.00000

Table 2 shows that compared with the other DMUs, DMU_{11} is efficient and presents a cross-efficiency score of 1 for most cases. Therefore, we choose DMU_{11} as the selected *CRP* and then rescale the cross-efficiencies of all DMUs according to

(10). The newly obtained cross-efficiency scores $\theta_{jk}^{11}\left(j,k\in[1,14]\right)$ of all DMUs are given in Table 3. From Table 3, we can observe that the new cross-efficiency scores θ_{jk}^{11} of DMU_{11} are all equal to 1.

According to (11), the *ECD*s of all DMUs, $c_k\left(k=1,2,\cdots,n\right)$, are calculated and outlined in the last row of Table 3. Moreover, the geometric mean values $\overline{\theta_j}\left(j=1,2,\cdots,n\right)$ of the rescaled cross-efficiency scores $\theta_{jk}^{r}\left(j=1,2,\cdots,n;k=1,2,\cdots,n\right)$ of $\mathrm{DMU}_j\left(j=1,2,\cdots,n\right)$ are calculated according to (12), which are shown in the last column of Table 3.

Table 3. *The rescaled cross-efficiency matrix.*

DMUj	DMUk														$\overline{\theta}_j$
	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}	DMU_{11}	DMU_{12}	DMU_{13}	DMU_{14}	
DMU_1	0.86836	0.45013	1.45215	0.86836	0.8492	1.06976	0.81076	0.78813	0.70308	0.75116	0.86836	0.7713	0.8492	0.8492	0.82947
DMU_2	0.17189	0.33794	0.11007	0.17189	0.17351	0.05593	0.24789	0.27242	0.28083	0.20577	0.17189	0.20249	0.17351	0.17351	0.18196
DMU_3	0.88259	0.19416	2.21032	0.88259	0.88442	1.56137	0.7232	0.68331	0.62254	0.78461	0.88259	0.80717	0.88442	0.88442	0.81995
DMU_4	0.95809	0.42586	1.6409	0.95809	0.94131	1.57836	0.8228	0.78498	0.69914	0.81125	0.95809	0.83407	0.94131	0.94131	0.90499
DMU_5	0.96528	0.36582	2.33274	0.96528	1.00000	2.26353	0.77038	0.7359	0.77775	1.00000	0.96528	1.00000	1.00000	1.00000	0.98114
DMU_6	0.88179	0.1108	2.23075	0.88179	0.87799	2.21051	0.66152	0.60837	0.50992	0.71759	0.88179	0.74779	0.87799	0.87799	0.77445
DMU_7	0.92108	0.77806	1.11335	0.92108	0.87951	0.76543	1.00000	1.00000	0.83947	0.78077	0.92108	0.80117	0.87951	0.87951	0.88649
DMU_8	0.78131	0.61137	1.20423	0.78131	0.77025	0.66178	0.84576	0.85876	0.82083	0.75318	0.78131	0.76308	0.77025	0.77025	0.78924
DMU_9	0.78545	0.72775	1.18398	0.78545	0.78885	0.60591	0.87824	0.90719	0.94774	0.83745	0.78545	0.83692	0.78885	0.78885	0.82329
DMU_{10}	0.78214	0.63539	1.52096	0.78214	0.82498	0.80672	0.77799	0.79436	1.00000	1.00000	0.78214	0.97192	0.82498	0.82498	0.86237
DMU_{11}	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
DMU_{12}	0.94619	0.63364	1.7496	0.94619	0.96020	0.99484	0.93618	0.93951	0.99984	1.00000	0.94619	1.00000	0.9602	0.9602	0.97752
DMU_{13}	1.00000	0.42565	2.33274	1.00000	1.00000	1.03107	1.00000	1.00000	1.00000	0.98429	1.00000	1.00000	1.00000	1.00000	1.00055
DMU_{14}	1.00000	0.22767	2.33274	1.00000	1.00000	2.26353	0.77950	0.72754	0.64784	0.85685	1.00000	0.88375	1.00000	1.00000	0.92475
c_k	0.8407	0.3807	0.2243	0.8407	0.8612	0.2793	0.7372	0.6908	0.6340	0.8068	0.8407	0.8475	0.8612	0.8612	0.8407

Using (13), the final efficiencies $A_j^*\left(j=1,\cdots,n\right)$ and the ranking orders are obtained as shown in Table 4. For the convenience of comparison, the ranking results based on the traditional CCR model as well as the ACE method, the BCE method, the NCE method, the CWE method are also outlined in Table 4. Obviously, the CCR model cannot discriminate the ranking orders of efficient DMUs, such as DMU_5, DMU_7, DMU_{10}, DMU_{11}, DMU_{12}, DMU_{13} and DMU_{14}. However, the other methods can fully rank the DMUs.

Table 4. *Efficiency scores and rankings of all DMUs with different methods.*

DMU	CCR		ACE method		BCE method		NCE method		CWE method		CEC-DEA method	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank
1	0.8684	12	0.5990	12	0.7516	12	0.7049	11	0.7021	11	0.8161	10
2	0.3379	14	0.1652	14	0.1897	14	0.1912	14	0.1698	14	0.1916	14
3	0.9475	11	0.6226	11	0.7681	9	0.7154	10	0.7117	9	0.8027	11
4	0.9581	9	0.6734	7	0.8198	6	0.7733	7	0.771	6	0.8827	7
5	1.0000	1	0.7983	1	0.8961	3	0.8764	2	0.8518	2	0.9384	4
6	0.9766	8	0.6385	9	0.7549	11	0.7024	12	0.6849	12	0.7589	13
7	1.0000	1	0.6478	8	0.8155	7	0.7711	8	0.7424	8	0.8852	6
8	0.8588	13	0.5855	13	0.7226	13	0.6906	13	0.6652	13	0.7834	12
9	0.9477	10	0.6309	10	0.7595	10	0.7378	9	0.7056	10	0.8192	9
10	1.0000	1	0.6813	6	0.7864	8	0.7813	6	0.7671	7	0.8460	8
11	1.0000	1	0.7742	2	0.9193	1	0.9041	1	0.9023	1	1.0000	1
12	1.0000	1	0.7314	5	0.8870	4	0.8541	4	0.842	3	0.9610	3
13	1.0000	1	0.7503	3	0.9190	2	0.8723	3	0.8348	4	0.9857	2
14	1.0000	1	0.7316	4	0.8659	5	0.8140	5	0.8001	5	0.8971	5

4.2. Model Validation Test

Some researchers have applied the multivariate statistics to conduct the validation test on DEA methods [34-36]. Using a method similar to that in [34], we test the credibility of the proposed CEC-DEA method via the multiple linear regression method. The final efficiencies are considered as dependent variables, and the inputs and outputs are considered as the independent variables. The multiple linear regression model is established as follows:

$$Efficiency=\beta_0+\beta_1\cdot Input\,1+\beta_2\cdot Input\,2+\beta_3\cdot Input\,3+\beta_4\cdot Output\,1+\beta_5\cdot Output\,2+\varepsilon \tag{14}$$

For comparison, we also test the credibility of both the CEC-DEA method and the traditional CCR model, the ACE method, the BCE method, the NCE method, the CWE method. Using SPSS software, we obtain the test results by employing the multiple linear regression model, and the results are shown in Tables 5 and 6. The results of our CEC-DEA model show higher consistency compared with the results of the other five methods. For example, the values of R, R square and the adjusted R square based on the proposed CEC-DEA model are 0.9860, 0.9723 and 0.9549, respectively, and all of these values are higher than the values obtained by the other five methods. In addition, the Analysis of Variance (ANOVA) shows that the CEC-DEA model passes the significance test, and its significance F value is 5.16074E-06, which is lower than that of the other five methods.

Table 5. *The summary of multiple linear regression model for methods.*

Method	Multiple R	R Square	Adjusted R Square	Std. Error of the Estimate
CCR model	0.9532	0.9086	0.8515	0.0673
ACE method	0.9773	0.9553	0.9274	0.04128
BCE method	0.9832	0.9667	0.9459	0.0422
NCE method	0.9831	0.9677	0.9475	0.039799
CWE method	0.9855	0.9713	0.9533	0.03779
CEC-DEA method	0.9860	0.9723	0.9549	0.0420

Table 6. *The ANOVA of multiple linear regression model for methods.*

Method		DF	SS	MS	F	p-value
	Regression	5	0.3600	0.0720	15.9033	0.000562
CCR model	Residual	8	0.0362	0.0045		
	Total	13	0.3962			
	Regression	5	0.291353	0.058271	34.19505	3.41E-05
ACE method	Residual	8	0.013633	0.001704		
	Total	13	0.304985			
	Regression	5	0.4131	0.0826	46.4753	1.06E-05
BCE method	Residual	8	0.0142	0.0018		
	Total	13	0.4273			
	Regression	5	0.37946	0.075892	47.91251	9.46E-06
NCE method	Residual	8	0.012672	0.001584		
	Total	13	0.392132			
	Regression	5	0.386172	0.077234	54.07414	5.94E-06
CWE method	Residual	8	0.011426	0.001428		
	Total	13	0.397598			
	Regression	5	0.4942	0.0988	56.0941	5.16074E-06
CEC-DEA method	Residual	8	0.0141	0.0018		
	Total	13	0.5083			

4.3. Results Discussion

As shown in Table 4, our proposed CEC-DEA method, the ACE method, the BCE method, the NCE method and the CWE method can fully rank the DMUs, whereas the CCR model cannot differentiate the efficient DMUs. In addition, the validation test results in section 4.2 demonstrate the credibility and effectiveness of the proposed CEC-DEA method.

A comparison of the ranking results of our CEC-DEA method with the results of the benevolent cross-efficiency method shows major variations, which are analyzed below.

With the benevolent cross-efficiency method, the final efficiencies of DMU_5 and DMU_{12} are 0.8961 and 0.8870, respectively, and their ranking orders are 3 and 4, respectively. However, with the CEC-DEA method, we obtain the opposite ranking order for DMU_5 and DMU_{12} and their efficiencies are 0.9384 and 0.9610, respectively.

With the benevolent cross-efficiency method, the final efficiencies of DMU_4 and DMU_7 are 0.8198 and 0.8155, respectively, and their ranking orders are 6 and 7, respectively. However, with the CEC-DEA method, we obtain the opposite ranking orders for DMU_4 and DMU_7 and their efficiencies are 0.8827 and 0.8852, respectively.

The ranking orders of DMU_3 and DMU_6 drop from 9 and 11 in the benevolent cross-efficiency method to 11 and 13 in the CEC-DEA method, respectively.

The main reason for the above changes is that the CEC-DEA method considers the *ECD*s of all DMUs and adopts the weighted geometric mean method to aggregate the cross-efficiency scores. Specifically, we use DMU_5 and DMU_{12} as examples to discuss the mechanism of the CEC-DEA method.

From Table 2 and Table 3, we observe that the cross-efficiency scores of DMU_5 are higher than those of DMU_{12} under the evaluation of DMU_1, DMU_4, DMU_5, DMU_{11}, DMU_{13}

and DMU_{14}, which present minor gaps, and DMU_3 and DMU_6, which present relatively larger gaps. However, under the evaluation of DMU_2, DMU_7, DMU_8 and DMU_9, the cross-efficiency scores of DMU_5 are lower than those of DMU_{12}.

In the last row of Table 3, we observe that DMU_3 and DMU_6 have the lowest *ECD*s, while DMU_1, DMU_2, DMU_4, DMU_5, DMU_7, DMU_8, DMU_9, DMU_{11}, DMU_{13} and DMU_{14} have relatively high *ECD*s.

Because of the *ECD*s, the peer-evaluations of DMU_3 and DMU_6 play a minor role, whereas the peer-evaluations of DMU_1, DMU_2, DMU_4, DMU_5, DMU_7, DMU_8, DMU_9, DMU_{11}, DMU_{13} and DMU_{14} play a relatively major role. Therefore, DMU_{12} acquires a higher final efficiency score than DMU_5 and ranks before DMU_5.

For clear illustration, we also present the cross-efficiency scores of DMU_5 and DMU_{12} as well as the *ECD*s of all DMUs in Figure 1.

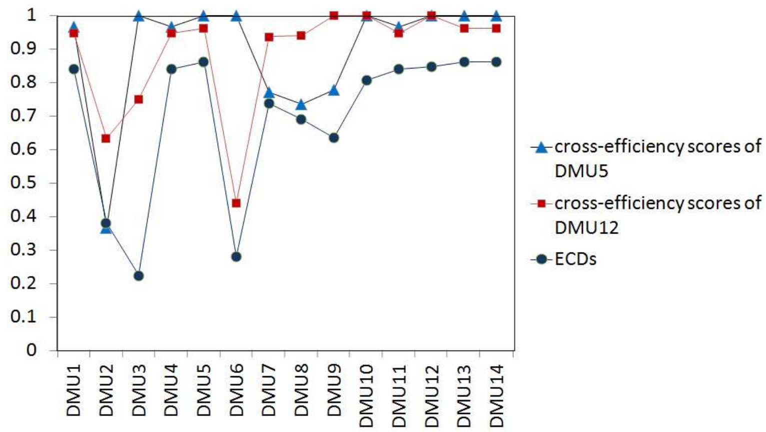

***Figure 1.** The cross-efficiency scores of DMU_5 and DMU_{12} and the ECDs of all DMUs.*

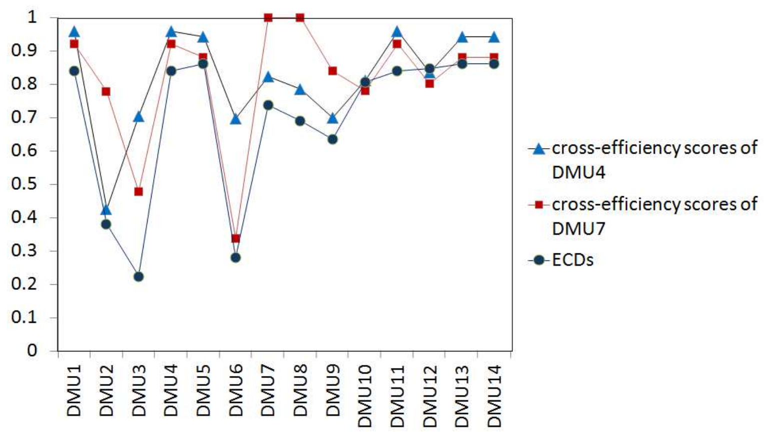

***Figure 2.** The cross-efficiency scores of DMU_4 and DMU_7 and the ECDs of all DMUs.*

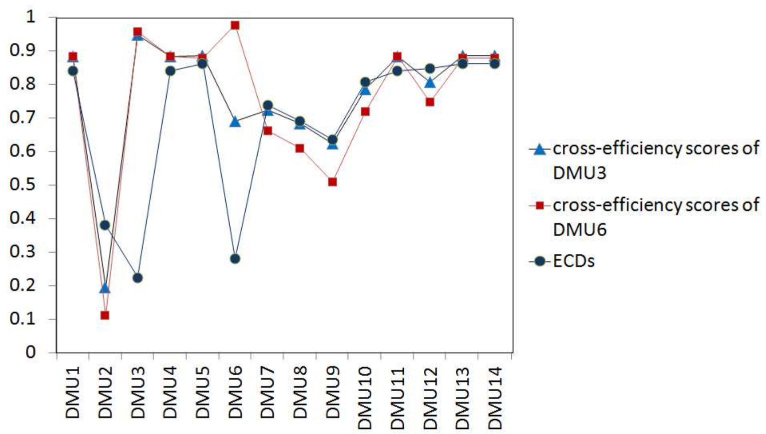

***Figure 3.** The cross-efficiency scores of DMU_3 and DMU_6 and the ECDs of all DMUs.*

Similarly, we can explain why DMU_4 ranks behind DMU_7 in the CEC-DEA method. For the convenience of analysis, we present the cross-efficiency scores of DMU_4 and DMU_7 as well as the *ECD*s of all DMUs in Figure 2. In addition, Figure 3 shows the cross-efficiency scores of DMU_3 and DMU_6 and the *ECD*s of all DMUs according to the proposed CEC-DEA method. Obviously, it is observed in Figure 3 that even though the self-evaluated efficiency scores of DMU_3 and DMU_6 are the highest, the *ECD*s of DMU_3 and DMU_6 are the lowest, which will greatly pull their orders backward in the final ranking result.

5. Conclusions

In the context of DEA, the cross-efficiency method has been extended and widely applied for evaluating DMUs in various areas. However, previous cross-efficiency methods do not consider the evaluation consensus among all DMUs, and most of them aggregate the cross-efficiency scores directly without considering the incomparability. Therefore, these methods cannot ensure the acceptance or recognition of all DMUs and cannot ensure a sufficiently reasonable evaluation. To resolve these problems, we propose a new DEA cross-efficiency method based on consensus (CEC-DEA), which consists of the following main parts: 1) a second-objective model is introduced to ensure the uniqueness of weights, which minimizes the total variance between the self-evaluated efficiencies and the peer-evaluated efficiencies based on the concept of consensus; 2) a DMU is selected as the *CRP* to ensure that the cross-efficiency scores are comparable, and all cross-efficiency scores are rescaled based on this DMU before aggregation; and 3) the *ECD* of each DMU is defined based on the concept of consensus, and the rescaled cross-efficiency scores are aggregated via the weighted geometric mean method, with the *ECD*s as the weights. Finally, a numerical example is presented, and the proposed CEC-DEA method is demonstrated to be effective.

However, our proposed DEA cross-efficiency method is based on the traditional radial DEA models, which have a problem with overestimating the efficiencies of DMUs when there exist some non-zero slacks. In the future, we can reserch a Non-radial DEA cross-efficiency method based on the slacks-based model (SBM) and DDF to overcome the above problem.

The Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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